CONCURSUL NAŢIONAL DE MATEMATICĂ APLICATĂ „ADOLF HAIMOVICI”

Barem de corectare

clasa a XI – a

Filiera teoretică - Profil real - Specializarea Ştiinţe ale naturii

1.

a) $A(x)\cdot A(y)=A(x+y), x,y\in\mathbb{R}$ 2p

b) $\det\left(A(x)\right)=1\neq 0,(\forall)x\in\mathbb{R}\Rightarrow(\exists)A^{-1}(x),(\forall)x\in\mathbb{R}$ 1p

din a) $\Rightarrow A(x)\cdot A(-x)=A(-x)\cdot A(x)=A(0)=I_3\Rightarrow A^{-1}(x)=A(x),(\forall)x\in\mathbb{R}$ 2p

c) din a) $\Rightarrow A(1)\cdot A(2)\cdot\ldots\cdot A(2015)=A(1+2+\ldots+2015)=A(2015\cdot 1008)$ 2p

2.

a) $A=\begin{pmatrix} a_{11} & a_{12} & a_{13}\\ a_{12} & a_{11} & a_{23}\\ a_{13} & a_{23} & a_{11}\end{pmatrix}$;

$a_{11}+a_{12}+a_{13}=a_{12}+a_{11}+a_{23}=a_{13}+a_{23}+a_{11}=k\Rightarrow a_{12}=a_{13}=a_{23}=\dfrac{k-a_{11}}{2}=\alpha$ 2p

exemple: $A=\begin{pmatrix} 1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}; B=\begin{pmatrix} -10 & 12 & 12\\ 12 & -10 & 12\\ 12 & 12 & -10\end{pmatrix}$ 2p

b) $\det A=\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{12} & a_{11} & a_{23}\\ a_{13} & a_{23} & a_{11}\end{vmatrix}=\begin{vmatrix} k & k & k\\ \alpha & a_{11} & \alpha\\ \alpha & \alpha & a_{11}\end{vmatrix}=k\cdot\begin{vmatrix} 1 & 1 & 1\\ \alpha & a_{11} & \alpha\\ \alpha & \alpha & a_{11}\end{vmatrix}=k\left(a^2_{11}+\alpha^2+\alpha^2-\alpha\cdot a_{11}-\alpha^2-\alpha\cdot a_{11}\right)=$

$k\left(a_{11}-\alpha\right)^2\Rightarrow\dfrac{1}{k}\cdot\det(A)=\left(a_{11}-\alpha\right)^2$

.... 3p

3.

a) $\lim\limits_{x\to\infty}x\left(\sqrt{x^2+ax+b}+cx\right)=2015\Rightarrow\begin{cases} c<0\\ \lim\limits_{x\to\infty}\dfrac{x\left(x^2+ax+b-c^2x^2\right)}{\sqrt{x^2+ax+b}-cx}=2015\Rightarrow\end{cases}$

$\lim\limits_{x\to\infty}\dfrac{\left(1-c^2\right)x^2+ax+b}{\sqrt{1+\dfrac{a}{x}+\dfrac{b}{x^2}}-c}=2015\Rightarrow\begin{cases} 1-c^2=0\Rightarrow c=-1\\ a=0\\ b=4030\end{cases}$ 4p

b)

$\lim\limits_{x\to 0}\left(\dfrac{2^x+3^x+\ldots+2015^x}{2014}\right)^{\frac{1}{x}}=\lim\limits_{x\to 0}\left[\left(1+\dfrac{2^x+3^x+\ldots+2015^x-2014}{2014}\right)^{\frac{2014}{2^x+3^x+\ldots+2015^x-2014}}\right]^{\frac{2^x+3^x+\ldots+2015^x-2014}{2014x}}=$

$= e^{\lim\limits_{x\to 0}\left(\frac{2^x+3^x+...+2015^x-2014}{2014x}\right)} = e^{\lim\limits_{x\to 0}\frac{1}{2014}\left(\frac{2^x-1}{x}+\frac{3^x-1}{x}+...+\frac{2015^x-1}{x}\right)} = e^{\frac{1}{2014}(\ln 2+\ln 3+....+\ln 2015)} =$ $= e^{\frac{1}{2014}\ln 2015!} = e^{\ln\left(\sqrt[2014]{2015!}\right)} = \sqrt[2014]{2015!}$ ……………………………………………………………………………………… … 3p
4.
a) f continua pe $(-\infty,4) \Rightarrow f$ continua şi în $x=2 \Leftrightarrow l_s = l_d = f(2) = a\cdot\log_2 2 = a$ …………..1p $l_s = a$, $l_d = -\frac{5}{3} \Rightarrow a = -\frac{5}{3}$ ……………………………………………………………………….2p
b) in relatia data înlocuim succesiv x cu $\frac{x}{2},\frac{x}{2^2},...,\frac{x}{2^n}$, insumăm relaţiile obţinute si avem : $f(x)-f\left(\frac{x}{2^{n+1}}\right)=x\left(1+\frac{1}{2}+...+\frac{1}{2^n}\right)=x\frac{1-\frac{1}{2^{n+1}}}{1-\frac{1}{2}}$, f continua în $x=0$ şi $f(0)=0$,trecând la   limită obţinem $f(x)-f(0)=2x,(\forall)x\in\mathbb{R}\Rightarrow f:\mathbb{R}\to\mathbb{R}, f(x)=2x$ ……………………… 4p